Beam-beam effects in crab crossing and crab waist scheme in KEKB

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Crossing angle

• Lorentz boost is used to make perpendicular field for moving direction. (J. Augustin, K. Hirata)
• Lorentz transformation seems to be not symplectic for the accelerator coordinate system $p_x = p_x/p_0$, remember adiabatic damping.
• Lorentz transformation is symplectic in the physical coordinate system.
Crossing angle

• Transformation from Lab. frame to head-on frame.

\[ x^* = \tan \theta z + [1 + h_x^* \sin \theta] x \]

\[ p_x^* = (p_x - h \tan \theta) / \cos \theta \]

\[ y^* = y + h_x^* \sin \theta x \]

\[ p_y^* = p_y / \cos \theta \]

\[ z^* = z / \cos \theta + h_z^* \sin \theta x \]

\[ p_z^* = p_z - p_x \tan \theta + h \tan^2 \theta \]

\[ h = p_z + 1 - \sqrt{(p_z + 1)^2 - p_x^2 - p_y^2} \]

(\( \theta \): half crossing angle)

Linear part

\[
\begin{pmatrix}
1 & 0 & 0 & 0 & \tan \theta & 0 \\
0 & 1 / \cos \theta & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 / \cos \theta & 0 & 0 \\
0 & 0 & 0 & 0 & 1 / \cos \theta & 0 \\
0 & -\tan \phi & 0 & 0 & 0 & 1
\end{pmatrix}
\]

Jacobian matrix and determinant of linear matrix contain \( 1 / \cos^3 \theta \) due to Lorentz transformation.

This transformation is symplectic.
Crab crossing scheme

- Transformation due to Crab cavity

\[ H_{crb} = -K_\phi x z \delta (s - s_{crb}) \]

- This transformation is expressed at IP

\[
\begin{pmatrix}
   x \\
   p_x \\
   z \\
   p_z
\end{pmatrix}
= 
\begin{pmatrix}
   1 & 0 & 0 & 0 \\
   0 & 1 & K_\phi & 0 \\
   0 & 0 & 1 & 0 \\
   K_\phi & 0 & 0 & 1
\end{pmatrix}\begin{pmatrix}
   x \\
   p_x \\
   z \\
   p_z
\end{pmatrix}
\]

- This transformation can be cancel the crossing angle effect.

\[ M_{IP} = M_{IP \leftarrow Crb} e^{-: H_{crb} :} M_{Crb \leftarrow IP} \\
= M_{IP \leftarrow Crb,0} e^{-: H_{crb} :} M_{IP \leftarrow Crb,0}^{-1} M_{IP,0} \]

\[ H_{crb, IP} = -\theta p_x z \delta (s - s^*) \]
Super-bunch collision scheme

Large Piwinski angle (K. Takayama et al.)

- Crab crossing: High current, high beam-beam parameter.

Super-bunch scheme in ee colliders: Low emittance, low beta, low current, so-called super bunch collision

P. Raimondi et al.
Tune shift

**Short bunch**

\[
\sqrt{\frac{\varepsilon_x \beta_x}{\sigma_z}} \geq 1
\]

\[
L \sim \frac{N^2}{\sqrt{\varepsilon_x \beta_x \varepsilon_y \beta_y}} \sim \frac{N \xi_y}{\beta_y}
\]

\[
\xi_x \sim \frac{N}{\varepsilon_x}
\]

\[
\xi_y \sim N \sqrt{\frac{\beta_y}{\varepsilon_x \beta_x \varepsilon_y}}
\]

\[
\beta_y > \sigma_z
\]

**Super-bunch (LPA)**

\[
\sqrt{\frac{\varepsilon_x \beta_x}{\sigma_z}} < 1
\]

\[
L \sim \frac{N^2}{\theta \sigma_z \sqrt{\varepsilon_y \beta_y}} \sim \frac{N \xi_y}{\beta_y}
\]

\[
\xi_x \sim \frac{N \beta_x}{\theta^2 \sigma_z^2}
\]

\[
\xi_y \sim \frac{N}{\theta \sigma_z} \sqrt{\frac{\beta_y}{\varepsilon_y}}
\]

\[
\beta_y > \frac{\sqrt{\varepsilon_x \beta_x}}{\theta}
\]
Crab waist scheme (P. Raimondi et al.)

\[ M = e^{-H_1} : M_0 e^{H_1} : \]

\[ H_1 = a x p_y^2 \]

\[ y = y + \frac{\partial H_1}{\partial p_y} = y + a x p_y \quad \quad p_x = p_x - \frac{\partial H}{\partial x} = p_x - a p_y^2 \]

- Take linear part for \( y \), since \( x \) is constant during collision.

\[
\begin{pmatrix}
\bar{\beta} & -\bar{\alpha} \\
-\bar{\alpha} & \bar{\gamma}
\end{pmatrix}
T
\begin{pmatrix}
\beta & -\alpha \\
-\alpha & \gamma
\end{pmatrix}T' =
\begin{pmatrix}
\beta + \frac{a^2 x^2}{\beta} & \frac{ax}{\beta} \\
\frac{ax}{\beta} & \frac{1}{\beta}
\end{pmatrix}
\]

\[ T = \begin{pmatrix}
1 & ax \\
0 & 1
\end{pmatrix} \]
\[
M(s) \left( \begin{array}{c}
\beta + \frac{a^2 x^2}{\beta} & \frac{ax}{\beta} \\
\frac{ax}{\beta} & \frac{1}{\beta}
\end{array} \right) M'(s) = \left( \begin{array}{c}
\beta + \frac{(s + ax)^2}{\beta} & \frac{s + ax}{\beta} \\
\frac{s + ax}{\beta} & \frac{1}{\beta}
\end{array} \right)
\]

\[
\beta \text{ waist is shifted to } s = -ax
\]

Taking \( a = 1/2\theta \)

- Beam particles with various \( x \) collides with other beam at their waist.

Beam shape is not Gaussian, but is like triangle at IP.

Beam shape on red beam frame
Nonlinear effect in the collision schemes

- Taylor map analysis is performed to study nonlinear characteristics of the collision scheme.

- Calculate beam-beam map

\[ \mathbf{x} = \mathbf{f}(\mathbf{x}_0) \quad \text{erf}(z + z_0) = \text{erf}(z_0) + \frac{2}{\sqrt{\pi n!}} \sum_{n=1}^{\infty} (-1)^{n-1} H_{n-1} e^{-z_0^2} z^n \]

- Remove linear part

\[ \mathbf{X} = \mathbf{f}(R^{-1}\mathbf{x}_0) = \mathbf{x}_0 + \sum a_{ij} x_{0,i} x_{0,j} + 3\text{-rd order} \ldots. \]

- Factorization, integrate polynomial

\[ \mathbf{X} = \exp\left( - : (H_3 + H_4 + \ldots) : \right) \mathbf{x}_0 \quad \sum a_{ij} x_{0,i} x_{0,j} = [-H_3, \mathbf{x}_0] \]
Coefficients of beam-beam Hamiltonian

- Expression-1 \((k_x, k_p, k_y, k_q, k_z, k_e)\) \(p=p_x, q=p_y, e=p_z\)
- Expression-2 \((n_x, n_y, n_z)\)

- 4-th order coefficients
  C400 \((400000), (310000), (220000), (130000), (040000)\)  
  C301 \((300010), (210010), (120010), (030010)\)  
  C220 \((202000), (112000), (022000), (201100), (111100), (021100), (200200), (110200), (020200)\)  
  C040 \((004000), (003100), (002200), (000300), (000400)\)  
  C121 \((102010), (012010), (101110), (011110), (100210), (010210)\)

- 3\(^{rd}\) order coefficients (except for chromatic terms)
  C300 \((300000), (210000), (120000), (030000)\)  
  C210 \((201000), (111000), (021000), (200100), (110100), (020100)\)  
  C120 \((102000), (012000), (101100), (011100), (100200), (010200)\)

- Low order nonlinear terms are efficient in e+e- colliders, while higher order terms are efficient in proton colliders.
Coefficient of $x^4$ and $y^4$

- Coefficient of $x^4$ decreases for a large Piwinski angle ($\theta\sigma_z/\sigma_x >> 1$).
Coefficient of $x^3$ and $x^3z$

- $x^3$ and $x^3z$ terms increase for $\theta\sigma_z/\sigma_x < 1$, then decrease at $\theta\sigma_z/\sigma_x >> 1$.
- These terms drive $n/3$ and side band resonances.
xy^2 and related terms

- These terms drive resonances $\nu_x + 2\nu_y = n$.
- Crab waist sextupole has this nonlinearity.
Tune scan with weak-strong simulation (KEKB)

$\sigma_y$ in tune space

$\theta = 0$

$\theta = 11$ mrad

Luminosity in tune space.
Crab waist scheme
compensation of nonlinear term with crab sextupole, KEKB

- $H = K \times p_y^2/2$, theoretical optimum, $2K\theta=1$.
- Clear structure- 220, 120, 121
- Flat for sextupole strength- 400, 301, 040
Tune scan with weak-strong simulation in crab waist scheme

\[ \theta \sigma_z / \sigma_x \sim 1 \]

\[ \theta \sigma_z / \sigma_x >> 1 \]

\( (0.5, 0.8) \)

\( (0.5, 0.5) \)

\( (0.8, 0.5) \)

- Crab waist gives good performance in simulations, though \( v_x + 2v_y = n \) line is seen.
- \( 2/3 \) resonance disappear for \( \theta \sigma_z / \sigma_x >> 1 \).
Measurements and Simulations in KEKB crab crossing
Measured luminosity in KEKB

\[ \xi_{y0H}^* \xi_{y0L} = 0.05 \]

beam-beam simulation
\( (I_b L/I_b H = 7/4) \)
Contents

• Simulation of knob scan with strong-strong simulation
• Beam-beam halo and luminosity with weak-strong simulation in SAD
• Touschek life time under the beam-beam interaction.
Simulation of knob scan
M. Tawada

- Current 0.8/1.4 mA/bunch (HER/LER)
- $\varepsilon_x = 24/18$ nm (HER/LER) 1% coupling
- $\beta_{x/y} = 80/0.7$ cm (both)
- $\nu_{x/y/z} = 0.511/0.580/0.025$

- R1-4, $\eta_y$, $\eta_{\perp y}$ of HER and LER (12 parameters) are scanned everyday in KEKB. The scan process was simulated using strong-strong simulation.
- Down hill simplex method for the 12 parameters is also used to optimize the luminosity.
HER r4, example I
HER ey, example 2
Example: Down hill Simplex after 2nd cycle with HER_r1, HER_r4, LER_r1, LER_r4

$L_{3,\text{simp}} = 23.7 \times 10^{30} \text{ cm}^{-2}\text{s}^{-1}/\text{bunch}$

Target: $L = 25 \times 10^{30} \text{ cm}^{-2}\text{s}^{-1}/\text{bunch}$
Summary for knob scan simulation

- R1 and R4 mean rotation of real and momentum space, respectively.
- R1(L)=R4(L)=R1(H)=R4(H) (others=0) means simple rotation of both beam should result no luminosity degradation.
- R1-R4 was not resolved and dispersion was mislead due to error each other in 1st cycle.
- The dispersion error was corrected, and R1-R4 tend to coincide for both ring at 2nd cycle.
- Tolerance of R4 is rough than that of R1.
- Regular scan does not seem to have problem.
- Simplex method also gave high luminosity.
Beam-beam life time
Simple arc transformation using matrix trans.

H-axis 0-12.8σₓ (0.1σₓ/unit)  V-axis 0-64σᵧ (0.5σᵧ/unit)

Contour plot with log scale

• The hor. and ver. halo do not matter for the operation.
• Vertical halo is formed by horizontal offset of collision.
• Halo is bigger in finite crossing angle collision compare than crab crossing.
iSize, beam size control of colliding beam using vertical dispersion (N. Iida), should work to suppress the vertical halo. Experiments showed iSize did not work for lifetime improvement.

\[ \varepsilon_y \quad 0.6 \times 10^{-10} \quad 1.8 \times 10^{-10} \quad 5.4 \times 10^{-10} \]

H-axis 0-12.8\(\sigma_x\) (0.1\(\sigma_x\)/unit)  V-axis 0-64\(\sigma_y\) (0.5\(\sigma_y\)/unit)
iSize scan $\sim$ V emittance scan

HER/LER V size vs. life

HER iSize bump 0.4 -> 0 mm

- In HER, longer beam life time is realized at larger vertical beam size for the iSize scan.
- In LER, life shortened due to a small beam size, which respond HER large beam size.

![Graphs showing HER and LER data over time](image)
Summary of halo simulation

• Simulation for halo formation is performed with a weak-strong simulation based on Gaussian model.
• The halo did not seem to affect the beam Life time.
• Life time asymmetry was not seen.
• Simulation gives that halo and beam size have positive correlation, but experiment did not show.
• Intrabeam effect? (Oide, Ohnishi)
Intrabeam scattering

- Cross-section of e-e scattering

\[
\frac{d\sigma}{d\Omega} = \frac{4r_e^2}{\gamma^2 p_\perp^4} \left[ \frac{4}{(1 - \sin^2 \theta \cos^2 \varphi)^2} - \frac{3}{1 - \sin^2 \theta \cos^2 \varphi} \right]
\]

- Touschek life

\[
\frac{1}{\tau} = 2N \int d\Omega \int dx_1 \int dx_2 A(x_1, x_2, \theta, \varphi) \frac{d\sigma}{d\Omega} v_\perp \psi(x_1) \psi(x_2) \]

\[
\int dx \psi(x) = 1
\]

- A=1 or 0 for outside/inside of aperture.
Monte Carlo integration for Touschek life time using SAD beam-beam tracking

- Distribution of $p$ is realistic one ($\rho_p$).
  \[
  \rho_p(p) = \sum_{i=1}^{n} \delta(p - p_i)
  \]
- Uniform distribution for $\Delta \delta < \delta_{\text{bucket}}$. The integral $\Delta \delta > \delta_{\text{bucket}}$ is given.
  \[
  f(\delta) = \sum_{i=1}^{n} \delta(\Delta \delta - \Delta \delta_i)
  \]

\[
\frac{1}{\tau} = \frac{32N\pi cr_e^2}{\gamma^3 V} \Delta \delta_{\text{bucket}} \sum_{i=1}^{n} A(p_{\perp,i},\Delta \delta_i) \frac{1}{p_{\perp,i}} \left[ \frac{2\gamma^3 p_{\perp,i}^3}{\Delta \delta_i^3} - \frac{\gamma p_{\perp,i}}{\Delta \delta_i} \right]
\]

\[
p_{\perp} > \frac{\Delta \delta}{\gamma}
\]
Results of Touschek life time
Preliminary

• Equilibrium beam distribution is formed by tracking several 1000 revolutions.

• Intrabeam kicks \((p, \Delta \delta)\) are applied randomly and \(A(p, \Delta \delta)\) of every macro-particles, which characterize inside or outside of aperture, are evaluated, and the life time integral is evaluated.

• The simulation gives a short life time in LER, \(\sim 53\) min.

• HER, \(\sim 297\) min.

• HER seems to have a sufficient life time.
Luminosity simulation with SAD weak-strong model

- Lattice nonlinearity may affect luminosity performance.
- Several signs in which nonlinearity affects luminosity.
  1. Existence of Golden orbit
  2. Luminosity performance depended on something run by run, even though IP parameters were tuned hardly.
  3. Integer part of tune affected the luminosity performance.
  4. Beta distortion in wiggler section made worse the luminosity.
Lspec given by SAD weak-strong

- Errors (sextupole position error) are generated and induced linear coupling and dispersion at IP are corrected and beam-beam tracking is executed in SAD.

Lattice nonlinearity does not affect strongly, if it is estimated correctly.
Summary for present KEKB

• Simulations have not given to explain the bad luminosity performance at KEKB yet.

• It is not clear whether the luminosity and life time issues are caused by an origin or independent two origins.

• New knobs for correction of lattice nonlinearity may be necessary, though present simulation results showed “no” now.

•
Super bunch collision scheme in KEKB (LPA, Italian option)

• Squeezing $\beta(\varepsilon)$ extremely, super-bunch collision is realized (P. Raimondi et al.)
• $\xi_x$ does not matter in super-bunch collision.
• Squeezing $\beta_x$, $\beta_y$ at IP gives high luminosity and low beam-beam parameter, while squeezing $\varepsilon_x$, $\varepsilon_y$ gives high luminosity and high beam-beam parameter.
• Increased beam-beam parameter at low $\varepsilon$ is compensated by lower current.
• Squeezing $\beta$ is essential for super-bunch collision.
• IR design is key issue.
Tune shift

Short bunch

\[
L \sim \frac{N^2}{\sqrt{\epsilon_x \beta_x \epsilon_y \beta_y}} \sim \frac{N \xi_y}{\beta_y}
\]

\[
\xi_x \sim \frac{N}{\epsilon_x}
\]

\[
\xi_y \sim N \sqrt{\frac{\beta_y}{\epsilon_x \beta_x \epsilon_y}}
\]

\[
\beta_y > \sigma_z
\]

Super-bunch (LPA)

\[
L \sim \frac{N^2}{\theta \sigma_z \sqrt{\epsilon_y \beta_y}} \sim \frac{N \xi_y}{\beta_y}
\]

\[
\xi_x \sim \frac{N \beta_x}{\theta^2 \sigma_z^2}
\]

\[
\xi_y \sim \frac{N \sqrt{\beta_y}}{\theta \sigma_z \sqrt{\epsilon_y}}
\]

\[
\beta_y > \frac{\sqrt{\epsilon_x \beta_x}}{\theta}
\]
Possible strategy at KEKB for super bunch scheme (my private opinion)

• Design of μ-β (β*=1cmx200μm) IP is the first priority.
• LER is possible to convert a low emittance lattice with addition of bending magnets and keeping vacuum chamber and other magnets (H. Koiso).
• High current scheme is not compatible with super-bunch scheme in HER.
• KEKB upgrade will be started from improvement of LER.
• We have chance to choose which scheme is adopted high current or super bunch, with watching the crab crossing and crab waist experiments.
parameters of several cases

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<th>Super KEKB</th>
<th>Normal ε</th>
<th>LER low-ε</th>
<th>L/H low-ε</th>
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Thank you
Traveling focus

\[ M = e^{-\cdot H_I} \cdot M_0 e^{\cdot H_I} \]

\[ H_I = \frac{a}{2} p_y^2 z \]

\[ \bar{y} = y + \frac{\partial H_I}{\partial p_y} = y + aP_y z \quad \bar{\delta} = \delta - \frac{\partial H_I}{\partial \gamma} = \delta - aP_y^2 \]

- Linear part for \( y, z \) is constant during collision.

\[
\begin{pmatrix}
\beta & -\alpha \\
-\alpha & \gamma
\end{pmatrix} = T \begin{pmatrix}
\beta & -\alpha \\
-\alpha & \gamma
\end{pmatrix} T' = \begin{pmatrix}
\beta + \frac{a^2 z^2}{\beta} & \frac{az}{\beta} \\
\frac{az}{\beta} & 1
\end{pmatrix}^{-1}
\]

\[ T = \begin{pmatrix}
1 & az \\
0 & 1
\end{pmatrix} \]

\[ \alpha = 0 \]
Crabbing beam in sextupole

- Crabbing beam in sextupole can give the nonlinear component at IP
- Traveling waist is realized at IP.

\[ H_I = \frac{a}{2} p_y^2 z \]

\[ z^* = \sqrt{\frac{\beta(s)}{\beta^*}} \zeta(s)x(s) \]

\[ K_2 \approx 30-50 \]

\[ K_2 = \frac{1}{2} \frac{B''L}{p/e} \approx \frac{1}{\theta \beta_y^* \beta_y} \frac{1}{\sqrt{\beta_x^*}} \sqrt{\beta_x} \]