Beam-Beam Resonances for Different Collision Schemes

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Introduction

One of the main advantages of proposed by P. Raimondi “Crab Waist” collision scheme is a strong suppression of betatron resonances excited by beam-beam interaction. In the Preprint LNF-07/003 (see also arXiv:physics/0702033) some qualitative explanations with numerical examples were given, describing beam-beam resonances for different collision schemes.

This presentation can be considered as an “appendix” (additional illustration) to that paper. We performed a number of full 2D betatron tune scans (beam-beam simulations) for different collision schemes, so one can easily see how the beam-beam resonances appear and disappear, depending on the colliding conditions.
Our main goal was to illustrate how the resonances excited by beam-beam interaction depend on the colliding conditions (hour-glass, crossing angle, Piwinski angle, Crab Waist). So, the most informative are comparisons of different pictures (scans) and the numbers of maximum luminosities. These comparisons can be not exact in terms of numerical values, but we believe that qualitatively they are quite relevant. Example:

**Head-on**

\[
L_{\text{max}} = 2.45 \cdot 10^{34}
\]

**Crossing angle**

\[
L_{\text{max}} = 2.05 \cdot 10^{34}
\]

“Geographical map” colors: red – “good”, blue – “bad”.
**Restrictions**

- **“Weak-Strong” simulations**
  It implies that in the “bad” working points the numbers are not correct. But we actually don’t need exact numbers in the “bad” areas. We need them in the “good” areas, where blowup is small and “weak-strong” approach works well.

- **Beam-Beam was the only nonlinearity**
  We studied “pure” beam-beam interaction without any other considerations (linear lattice).

- **Emittances are the input parameters, independent on the betatron tunes**
  In fact, it is impossible to achieve such a small vertical emittance near the main coupling resonance, but we did not care about this.

- **Parasitic Crossings were not considered**
  Luminosity was calculated for the same number of bunches (as for SuperB), just for comparison. But in case of head-on collision such a bunch spacing would be impossible…

- **Some others…**
  Lattice without coupling, simplified simulation of noises (without correlations), beam tails and lifetime are not considered, etc., etc.

  **If we want to perform a wide range tune scan we cannot avoid these restrictions…**
## Set of parameters ("nominal")

For the basis we took the SuperB set of parameters of 15.11.2006, electrons being the “strong” beam (7 GeV) and positrons – the “weak” one (4 GeV):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal beta</td>
<td>$\beta_x'$ (mm)</td>
</tr>
<tr>
<td>Vertical beta</td>
<td>$\beta_y'$ (mm)</td>
</tr>
<tr>
<td>Horizontal emittance</td>
<td>$\varepsilon_x$ (nm)</td>
</tr>
<tr>
<td>Vertical emittance</td>
<td>$\varepsilon_y$ (nm)</td>
</tr>
<tr>
<td>Bunch length:</td>
<td>$\sigma_z$ (mm)</td>
</tr>
<tr>
<td>Energy spread:</td>
<td>$\sigma_E$</td>
</tr>
<tr>
<td>Synchrotron tune (e+)</td>
<td>$\nu_s$</td>
</tr>
<tr>
<td>Damping decrements:</td>
<td>$\alpha_{x,y}$</td>
</tr>
<tr>
<td>Circumference</td>
<td>C (m)</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>$N_b$</td>
</tr>
<tr>
<td>Particles per bunch (e-)</td>
<td>$N_s$</td>
</tr>
<tr>
<td>Particles per bunch (e+)</td>
<td>$N_w$</td>
</tr>
<tr>
<td>Crossing angle (full)</td>
<td>$\theta$ (mrad)</td>
</tr>
<tr>
<td>Piwinski angle</td>
<td>$\phi$</td>
</tr>
<tr>
<td>“Nominal” tune shifts</td>
<td>$\xi_{x,y}$</td>
</tr>
<tr>
<td>“Actual” tune shifts</td>
<td>$\xi_{x,y}$</td>
</tr>
<tr>
<td>Luminosity</td>
<td>L</td>
</tr>
</tbody>
</table>

### Piwinski angle

$$\phi = \frac{\sigma_z}{\sigma_x} \tan \left( \frac{\theta}{2} \right)$$

### Basic relations for large Piwinski angle:

$$L \propto \frac{N \cdot \xi_y}{\beta_y}$$

$$\xi_y \propto \frac{N \cdot \beta_y}{\sigma_x \sigma_y \cdot \sqrt{1 + \phi^2}}$$

$$\xi_x \propto \frac{N}{\varepsilon_x \cdot (1 + \phi^2)}$$
Parameters for other simulations

- We tried to keep the “nominal” set of parameters as close as possible. However, for head-on and small crossing angle collisions we had to change some parameters in order to have acceptable tune shifts. The idea was to *keep the $\xi$ value close to the limit in the “good” areas*, in this case the picture of resonances will be the most clear and informative.

- **Collisions with changed parameters were not optimized for themselves.** We made only minimal changes to get the “correct” $\xi$ value.

- **“Optimistic” luminosity extrapolation.** Luminosity was calculated for the same number of bunches, without taking into account parasitic crossings (PC). This is quite reliable for large Piwinski angle, but for head-on collision the bunch spacing must be larger…
1) **“Ordinary conditions”: head-on and crossing with small Piwinski angle**

\[ \beta^*_{x,y} \Rightarrow 20 \cdot \beta^*_{x,y} \quad \varepsilon_{x,y} \Rightarrow 44 \cdot \varepsilon_{x,y} \]

“Ordinary conditions” mean that \( \beta^*_{y} \) must be increased by a factor of 20 to match the bunch length. Also, we decided to have the same \( \beta^*_{x} / \beta^*_{y} \) ratio, the same bunch length and bunch current. If the emittances would be also the same, the “nominal” \( \xi_{x,y} \) would not change as well. But we need to reduce them to acceptable values, let’s say \( \xi_{y}=0.07 \). To achieve this, we increased both emittances by a factor of 44. In this case \( \xi_{x}=0.0286 \), and the same crossing angle \( \theta = 34 \text{ mrad} \) results in Piwinski angle of \( \phi = 0.6 \) – again we have “ordinary conditions” here. We performed the following scans:

- Head-on collision, \( \beta^*_{y}=\sigma_{z} \) (normal conditions)
- Head-on collision, bunch length decreased by a factor of 100 (eliminates hour-glass effect) and increased by factors of 2 and 3 (enhanced hour-glass effect)
- Crossing angle, normal conditions (Piwinski angle \( \phi = 0.1, 0.2, 0.3, 0.6, 1.2, 1.8 \))

2) **Large Piwinski angle —> “Crab Waist”**

This new concept is based on three ideas: large Piwinski angle, very small \( \beta^*_{y} \) to match the overlapping area, and finally the “Crab Waist”. We investigated all these ideas step by step:

- Large vertical beta: \( \beta^*_{y}=\sigma_{z} \), without CW
- Small (nominal) vertical beta: \( \beta^*_{y}=\sigma_{z}/20 \), with and without CW

In this scheme X-Y betatron resonances are similar to synchro-betatron resonances due to hour-glass effect in head-on collisions. And “Crab Waist” kill them…
Head-on, $\beta_y = 6\text{mm}, \sigma_z = \beta_y$

Luminosity, $L_{\text{max}} = 2.45 \cdot 10^{34}$

Inverse Vertical Blowup

Inverse Horizontal Blowup

Resonances $L \cdot \nu_x + M \cdot \nu_y = k$ (L, M – even numbers)

Red: up to 4th order, Green: 5th – 6th orders

The X-Y betatron resonances appear due to the vertical beam-beam kick’s dependence on the horizontal particle’s coordinate (amplitude modulation). The horizontal kick also depends on the vertical coordinate, but for the flat beams this dependence is much weaker.
collision point (cp): the center of the opposite bunch.

βy dependence on the longitudinal shift of cp:
- luminosity reduction (geometrical factor)
- ξy increases when cp is shifted

synchro-betatron resonances due to:
- vertical betatron phase modulation at cp
- amplitude modulation (ξy dependence on βy at cp)

strong dependence on synchrotron tune (larger – worse)!

vertical betatron phase averaging over the interaction region:
- high-order vertical resonances suppression.
\[ \sigma_z = \beta_y / 100 \]

\[ \sigma_z = \beta_y \cdot 2 \]

\[ \sigma_z = \beta_y \cdot 3 \]

Luminosity, \( L_{\text{max}} = 3.17 \cdot 10^{34} \)

Luminosity, \( L_{\text{max}} = 1.98 \cdot 10^{34} \)

Luminosity, \( L_{\text{max}} = 1.62 \cdot 10^{34} \)

Inverse Vertical Blowup

Inverse Vertical Blowup

Inverse Vertical Blowup
Hour-glass: summary

• **Without hour-glass**
  - Luminosity increases (geometrical factor).
  - Resonance lines become thinner, since the synchro-betatron satellites disappeared.
  - More high-order resonances become visible, since the vertical betatron phase averaging disappears, so a particle fills a “solid” kick in a constant phase.

• **Enhanced hour-glass**
  - Luminosity decreases (geometrical factor).
  - Synchro-betatron resonances become stronger (wider resonance lines).
  - For strong hour-glass there are no working points without strong blowup.
  - Taking into account beam tails, the situation looks even worse…
Horizontal coordinate of the test particle in CP (in the strong bunch’s coordinate frame) now depends mainly on its longitudinal coordinate, that results in a strong amplitude modulation of both horizontal and vertical beam-beam kicks by the synchrotron oscillations, thus exciting strong synchro-betatron resonances.
Crossing angle: Luminosity vs. $\phi$

$\phi = 0.1, \quad L_{\text{max}} = 2.42 \cdot 10^{34}$

$\phi = 0.2, \quad L_{\text{max}} = 2.38 \cdot 10^{34}$

$\phi = 0.3, \quad L_{\text{max}} = 2.30 \cdot 10^{34}$

$\phi = 0.6, \quad L_{\text{max}} = 2.05 \cdot 10^{34}$

$\phi = 1.2, \quad L_{\text{max}} = 1.61 \cdot 10^{34}$

$\phi = 1.8, \quad L_{\text{max}} = 1.30 \cdot 10^{34}$
Crossing angle: summary

When increasing the Piwinski angle:

• **Luminosity and real tune shift decrease**
  Geometrical facor (we used the same bunch current for all these simulations).

• **Betatron resonances** \( \nu_x \pm 2 \nu_y = k \) **become stronger**
  Need crossing angle to break the X-symmetry.

• **The other betatron resonances become weaker**
  One reason – real tune shift decreasing (in these simulations).
  Another reason – horizontal coordinate of the test particle in CP (in the strong bunch’s coordinate frame) now depends more on its longitudinal coordinate and less on the horizontal betatron coordinate.

• **Synchro-betatron resonances become stronger**
  The “good” areas shrink.

**In general, it looks like the more Piwinski angle – the worser, but...**
Large Piwinski angle: new concept of CP

For large horizontal separations (in units of $\sigma_x$) the vertical kick drops as $1/R^2$, while the horizontal kick – as $1/R$. As we consider mainly the vertical motion, the center of the opposite bunch is not so important and can be not seen at all (due to large horizontal separation) by the particles with large longitudinal displacements. So, the CP has to be defined in a different way: now it is the point where a test particle crosses the longitudinal axis of the opposite beam. So, X-coordinate of CP in the “strong” frame is always zero, by the definition.
Vertical beam-beam kick’s dependence on the particle’s X-coordinate becomes very small, since the particle shifted horizontally crosses the opposite bunch in the point slightly shifted longitudinally, but with actually the same density, and geometry of collision will be the same as for the equilibrium particle. This makes the X-Y betatron resonances much weaker than even in the ordinary head-on collisions.
Low $\beta_y (\phi = 18, \beta_y = 0.3 \text{ mm}, \sigma_z \gg \beta_y)$, but CW = 0

One of the main differences between large and small Piwinski angles: horizontal synchro-betatron resonances are enhanced, while the vertical ones are suppressed (D. Pestrikov, [4]), as well as horizontal betatron ones.

Since the longitudinal shift of CP due to horizontal betatron oscillations is comparable now with $\beta_y$, the vertical betatron phase at CP and $\xi_y$ are strongly modulated by the horizontal betatron oscillations, thus exciting X-Y betatron resonances.

This is similar to resonances induced by hour-glass in head-on collisions.
Betatron resonances suppression by the Crab Waist

Crab Waist eliminates the vertical betatron phase modulation at the CP, induced by the horizontal betatron oscillations.
Amplitude modulation due to X-betatron oscillations

Without Crab Waist $\beta_{yw}$ and $\beta_{ys}$ at the CP are almost identical, so $\xi_y$ scales as $(\beta_{ys})^{+1/2}$.

With Crab Waist $\beta_{yw} = \text{const}$ at the CP, so $\xi_y$ scales as $(\beta_{ys})^{-1/2}$.

This means that if the Waist rotation is less than the nominal value, the amplitude modulation decreases, while some phase modulation appears again.

The optimum Waist rotation is usually in the range of 0.6 to 0.8. It is a compromise between amplitude and phase modulation, and depends on other parameters ($\xi$, $\phi$, etc.).
Crab Waist: OFF and ON (current is larger by 2.5 !)

If $\beta_{ys}$ is larger and $\varepsilon_{ys}$ is smaller by a factor of 100, there is no $\beta_{ys}$ modulation, while $\sigma_{ys}$ does not change.

These simulations, of course, are not realistic. The only goal is to demonstrate how the X-Y betatron resonances are suppressed by the Crab Waist.
Summary

Head-on, $L_{\text{max}} = 2.45 \cdot 10^{34}$

Ordinary crossing, $L_{\text{max}} = 2.05 \cdot 10^{34}$

Large $\phi$, CW = 0, $L_{\text{max}} = 1.6 \cdot 10^{35}$

Crab Waist, $L_{\text{max}} = 1.05 \cdot 10^{36}$

$\xi_y = 0.07$

$\xi_y = 0.17$
Acknowledgements

All these simulations were performed on lxcalc cluster (LNF-INFN, Frascati, Italy). Thanks to LNF Computing Division for their support!

References


Appendix

See all the luminosity contour plots in larger scale on the next pages.
Head-on, $\beta_y = 6\text{mm}$, $\sigma_z = \beta_y$
Head-on, $\beta_y = 6\text{mm}$, $\sigma_z = \beta_y / 100$
Head-on, $\beta_y = 6\text{mm}$, $\sigma_z = \beta_y \cdot 2$
Head-on, $\beta_y = 6\text{mm}$, $\sigma_z = \beta_y \cdot 3$
Crossing angle ("ordinary", $\phi = 0.1$), $\beta_y = 6\text{mm}$, $\sigma_z = \beta_y$
Crossing angle ("ordinary", $\phi = 0.2$), $\beta_y = 6\text{mm}$, $\sigma_z = \beta_y$
Crossing angle ("ordinary", $\phi = 0.3$), $\beta_y = 6\text{mm}$, $\sigma_z = \beta_y$
Crossing angle ("ordinary", $\phi = 0.6$), $\beta_y = 6\text{mm}$, $\sigma_z = \beta_y$
Crossing angle ("ordinary", $\phi = 1.2$), $\beta_y = 6\text{mm}$, $\sigma_z = \beta_y$
Crossing angle ("ordinary", $\phi = 1.8$), $\beta_y = 6\text{mm}$, $\sigma_z = \beta_y$
Small emittances and $\beta_x$, large Piwinski angle: $\phi = 18$, $\beta_y = 6\text{mm}$, $\sigma_z = \beta_y$
Nominal parameters ($\phi = 18$, $\beta_y = 0.3$ mm, $\sigma_z >> \beta_y$), but CW = 0
Nominal parameters, CW = 0, without $\beta_y$ modulation
Nominal parameters, CW = 1, without $\beta_{ys}$ modulation
Nominal parameters, CW = 1
Nominal parameters, CW = 0.8