Longitudinally polarized electrons in SuperB

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## Options for longitudinal spin

1. **Siberian snake (180° spin rotator).**
   - Spin lies in horizontal plane everywhere. Simplicity, Universality, but depolarization $\tau_\text{tau} \sim E^{-7}$.
   - Practical limit is $E < 2.5$ GeV for LER bending radii.
   - $E=2$ GeV
   - $B_l = \pi BR = 21$ T*m

2. **90° solenoids before FF-bends and after.**
   - FF horizontal bends rotate spin around y-axis from x- to z-direction. Spin is upright in arcs.
   - Scheme works only at certain energy. Two insertions are needed to get the spin transparency.
   - $E=4-7$ GeV
   - $B_l = (\pi/2)BR$

3. **HERA-like 90° spin rotator. Combination of transverse bends (J.Buon, K.Steffen).**
   - Upright spin in arcs.
   - Advantages: vertical bends are cheaper than solenoids. Spin transparency is garanteed.
   - Disadvantage: few meters of the vertical orbit excursion before/after FF.
   - $E=7$ GeV
Spin motion description

\[ \vec{V} = V \left( \vec{e}_z + x'\vec{e}_x + y'\vec{e}_y \right) \]

Orts of the co-moving frame:

\[ \vec{a}_1 = \frac{\vec{e}_y \times \vec{V}}{|\vec{e}_y \times \vec{V}|} \quad \vec{e}_x - x'\vec{e}_z \]

\[ \vec{a}_2 = \vec{a}_3 \times \vec{a}_1 \quad \vec{e}_y - y'\vec{e}_z \]

Spin perturbations:

\[ w_x \square v_0 \left( -K_x \frac{\Delta \gamma}{\gamma} - y'' \right) \]

\[ w_y \square v_0 \left( -K_y \frac{\Delta \gamma}{\gamma} + x'' \right) \]

\[ w_z \square \left( 1 + a \right) K_z \frac{\Delta \gamma}{\gamma} \]

Spin-orbit coupling terms

Chromaticity of spin rotation

\[ K_{x,y,z} = B_{x,y,z} / \langle B_y \rangle \]
Periodic closed spin orbit – always exist!

Derbenev, Kondratenko, Skrinsky, 1970

\[ \vec{n}(\theta + 2\pi) = \vec{n}(\theta) \]

Real orthogonal vectors \( \vec{\eta}_1, \vec{\eta}_2 \) describe precession of spin around \( \vec{n} \).

\[ \vec{\eta}_1 \times \vec{\eta}_2 = \vec{n} \]

Complex vectors: \( \vec{\eta} = \vec{\eta}_1 - i\vec{\eta}_2, \quad \vec{\eta}^* = \vec{\eta}_1 + i\vec{\eta}_2 \)

are more convenient to use for description of rotation by the angle \( \phi \) around \( \vec{n} \) direction:

\[ \vec{\eta}(\theta) = \vec{\eta}(0)e^{i\phi}, \quad \vec{\eta}(\theta)^* = \vec{\eta}(0)^*e^{-i\phi} \]

\[ \vec{\eta}(\theta + 2\pi) = \vec{\eta}(\theta)e^{i2\pi\nu} \] - precession around \( \vec{n} \) by the angle \( 2\pi\nu \).

\( \nu \) - is a spin tune. In the flat machine and without solenoids \( \nu = \nu_0 = \gamma a \).
Calculation of spin orbit distortions

\[ \Delta \tilde{n}(\theta) = \text{Re} \left( i \tilde{\eta}(\theta)^* \int_{-\infty}^{\theta} \tilde{w} \tilde{\eta} d\theta \right) \]

\[ \int_{-\infty}^{\theta} \tilde{w} \tilde{\eta} d\theta = \sum_i \int_{\theta-2\pi}^{\theta} \tilde{w}_i \tilde{\eta} d\theta = \sum_i \int_{\theta}^{\theta+2\pi} \tilde{w}_i \tilde{\eta} d\theta \]

\( \nu \) - is a spin tune

\( \nu_i = 0, \pm \nu_x, \pm \nu_y \) transverse motion frequencies, \( w_i(\theta + 2\pi) = w_i(\theta)e^{i2\pi \nu_i} \)
Calculation of a spin-orbit coupling vector

\[ \Delta \bar{n}(\theta) = \text{Re} \left( i \bar{n}(\theta)^* \int_{-\infty}^{\theta} \bar{w} \bar{n} d\theta \right) \]

\[ \vec{d} \equiv \frac{\Delta \bar{n}(\theta)}{\left( \frac{\Delta \gamma}{\gamma} \right)} \text{ - spin-orbit coupling vector} \]

\[ \vec{d} = \vec{d}_\gamma + \vec{d}_\beta \]

\[ \vec{d}_\gamma \text{ - direct dependence of } \bar{n} \text{ on the energy} \]

\[ \vec{d}_\beta \text{ - contribution from the betatron motion} \]

\[ \vec{d}_\gamma = 0 \text{ - zero spin chromaticity!} \]

\[ \vec{d}_\beta = 0 \text{ - spin transparency!} \]
Polarization Scenario for E=2 GeV

• Polarized electron source.
• Acceleration in a linac.
• About $5 \times 10^{10}$ electrons/pulse at about 40 Hz are needed to compensate particle losses caused by luminosity and by the Touschek effect. Estimation of a lifetime $\tau \sim 100$ s.
• Depolarization time much longer ($\tau \sim 4000$ s at E=2 GeV).
• Establish the closed spin orbit by placing Siberian Snake in the straight section opposite to IP (option for the LER).
• Spin at IP is directed longitudinally at any energy! Spin tune is half integer in case of full Snake and fractional with the Partial Snake.
• Rotation of spin by $180^0$ around z-axis is provided by the solenoid field integral $B I = \pi B R = 21$ Tm for E=2 GeV.
Spin orbit in presence of Siberian Snake

Derbenev, Kondratenko, Skrinsky, 1977

Snake rotates spin by $180^0$ around z-axis

In arcs spin lies in the horizontal plane

At IP the spin is directed longitudinally (exactly!)

With a partial snake at a magic energy spin is directed also longitudinally at IP as well at the snake’s location
180° Spin Rotator for Siberian Snake

Decoupling Optics: \( T_x = -T_y \)

\[
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\]

- for the spin transparency!

Each of two solenoids provide 90° spin rotation

All quads are not skewed!

Litvinenko, Zholentz, 1980
Spin Rotator for partial Snake

For decoupling again should be $T_x = -T_y$

$$T_x = \begin{pmatrix} -\cos \varphi & -2r \sin \varphi \\ (2r)^{-1} \sin \varphi & -\cos \varphi \end{pmatrix}$$ for the spin transparency!

Two solenoids provide spin rotation by $\varphi \leq 180^0$

All quads don’t need to be skewed!

$r = pc / eB$
90° Spin Rotator optics

Decoupling condition:  \( T_x = -T_y \)

\[
T_x = -T_y = \begin{pmatrix} 0 & -2r \\ (2r)^{-1} & 0 \end{pmatrix}
\]

for the spin transparency!

Each of two solenoids provide 45° spin rotation

\( r = \frac{pc}{eB} \)
Depolarization time in presence of Siberian Snake

\[ \tau_p^{-1} = \frac{5\sqrt{3}}{8} \lambda_e r_e c \gamma^5 \left\langle \frac{1 - \frac{2}{9} (\vec{n}\vec{v})^2 + \frac{11}{18} \vec{d}^2}{|r|^3} \right\rangle \]

\[ \vec{d} = \gamma \frac{\partial \vec{n}}{\partial \gamma} \] is the spin – orbit coupling vector

\[ \langle \vec{d}^2 \rangle_{\text{min}} = \frac{\pi^2}{3} \nu^2 \]

Betatron oscillations could increase \(|\vec{d}|\)!
Spin transparency for the snake is desirable.

For \( E = 2 \text{ GeV} \) (\( \nu = \gamma a = 4.54 \)), \( r = 20 \text{ m} \), \( \tau_p = 4000 \text{ s} \gg \tau_{\text{life}} \)

Equilibrium selfpolarization degree \( \zeta \parallel \vec{b}\vec{n} = 0!!! \) (Here \( \vec{b} = \vec{B}/B \))
Different polarization options

1. Single snake (full or partial): \( \langle \tilde{d}^2 \rangle = \pi^2 v_0^2 \left( \frac{1}{\sin^2 \varphi} - \frac{2}{3} \right) \)

   Full snake is preferable due to much lower \( \langle \tilde{d}^2 \rangle \)

2. Single insertion with two 90\(^0\) rotators and 180\(^0\) rotation around the vertical axis in between: \( \langle \tilde{d}^2 \rangle_{\text{min}} = \frac{\pi^2 v_0^2}{4} \)

   Bend in between two spin rotators:
   - 39.6\(^0\) for E=2 GeV
   - 19.8\(^0\) for E=4 GeV, 11.3\(^0\) for E=7 GeV

   Two such insertions placed in sequence compensate spin chromaticity of each other!
Different polarization options, cont’d

3a. Single insertion with +/-90° solenoids and anti-symmetric bend in between

![Diagram of a circular path with anti-symmetric bend and insertion points]

Advantage: Spin direction is achromatic in arcs. \(< d^2 >_{\text{arcs}} = 0\).
Disadvantage: Extra bends are needed.
Different polarization options, cont’d

3b. +/-90° solenoids and anti-symmetric bend in between (not in scale!).

Advantage: Less number of bends between spin rotators compared to 3a. version.
Spin direction is achromatic in arcs. \(< d^2 >_{\text{arcs}} = 0\).
Different polarization options, cont’d

3c. Two identical insertions with +/-90° solenoids and anti-symmetric bends in between.

Advantage: Spin direction is achromatic in arcs. \( < d^2 >_{\text{arcs}} = 0 \).
Two arcs become identical to each other. Second IP presented as option.
Disadvantage: Extra bends are needed.
Transverse bends $90^0$ spin rotator

All vertical and horizontal bends are equal to $5.66^0$ at 7 GeV ($90^0$ for spin). They are achromatic being divided in two half-bends and lenses in between.

After two first bends x-plane becomes inclined by 97.5 mrad. Could be rolled back by weak solenoid with $Bl=0.455 \, \text{T} \cdot \text{m}$.

Advantages: Dipoles are cheaper than solenoids; Spin transparent solution.

Comparison of Lifetimes

Beam bremsstrahlung cross-section:

\[ \sigma_{\text{Loss}} \approx 1.8 \cdot 10^{-25} \text{ cm}^2 \quad \text{- estimated by E. Paoloni} \]

For \( L = 10^{36} \text{ cm}^{-2} \text{s}^{-1} \), \( \dot{N} \approx 1.8 \cdot 10^{11} \text{ s}^{-1} \)

\[ \tau_{\text{Lum}} = \frac{2.4 \cdot 10^{14}}{1.8 \cdot 10^{11} \text{(s}^{-1})} = 1300 \text{ s} \]

\[ \tau_{\text{Touschek}} = 100 \text{ s?} \]

\[ \tau_p = 4000 \text{ s} \]

\[ \frac{\tau_p}{\tau_{\text{beam}}} \approx 40 \]
Polarization equilibrium.

Polarization degree of electrons from a gun: $\zeta_{beam} = 0.9$

Asymmetric FF bends ($d_{arc}=0$!). Spin relaxation time: $\tau_p = 3500$ s

Equilibrium polarization by SR: $\zeta_p = 0.12$ (4 GeV), $\zeta_p = 0.06$ (7 GeV)

Average polarization (taking into account some depolarization in a ring, mainly by wigglers): $\tau_{beam} = 3$ min.

\[
\zeta = \zeta_{beam} \frac{\tau_p}{\tau_{beam} + \tau_p} + \zeta_p \frac{\tau_{beam}}{\tau_{beam} + \tau_p}
\]

Finally average polarization with asymmetric FF bends:

$\zeta = 87\%$ (4 GeV), $\zeta = 84\%$ (7 GeV)
What one could gain having two polarized beams?

\[ \zeta = \frac{\zeta_1 + \zeta_2}{1 + \zeta_1 \zeta_2} \] - event rate asymmetry

An example: \( \zeta = 0.995 \) for \( \zeta_1 = \zeta_2 = 0.9 \)

\[ \frac{L}{L_0} = 1 + \zeta_1 \zeta_2 = 1.81 \] - gain in luminosity compared to the unpolarized beams

Polarized positrons? Difficult task!
Decoupling Insertion between two Solenoids

\[ M_{\text{Sol}} = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} \cdot \begin{pmatrix} I \cdot \cos(\varphi) & I \cdot \sin(\varphi) \\ -I \cdot \sin(\varphi) & I \cdot \cos(\varphi) \end{pmatrix} \]

\[ M_{\text{Sol}} \cdot \begin{pmatrix} T_x & 0 \\ 0 & T_y \end{pmatrix} \cdot M_{\text{Sol}} = ??? \quad \text{For } T_x = -T_y \rightarrow \]

\[ \begin{pmatrix} I \cdot \cos(\varphi) & I \cdot \sin(\varphi) \\ -I \cdot \sin(\varphi) & I \cdot \cos(\varphi) \end{pmatrix} \cdot \begin{pmatrix} T & 0 \\ 0 & -T \end{pmatrix} \cdot \begin{pmatrix} I \cdot \cos(\varphi) & I \cdot \sin(\varphi) \\ -I \cdot \sin(\varphi) & I \cdot \cos(\varphi) \end{pmatrix} = \]

\[ = \begin{pmatrix} T & 0 \\ 0 & -T \end{pmatrix} \rightarrow M_{\text{Sol}} \cdot \begin{pmatrix} T & 0 \\ 0 & -T \end{pmatrix} \cdot M_{\text{Sol}} = \begin{pmatrix} ATA & 0 \\ 0 & -ATA \end{pmatrix} \]
Compton scattering of circular polarized light on longitudinally polarized electrons – high asymmetry!
Conclusion

• High degree $\zeta > 90\%$ electron polarization is achievable from a gun.

• Siberian Snake concept is applicable below $2.5 \ \text{GeV}$. It provides the longitudinal spin direction at IP. Could be made spin transparent. But, unavoidably any snake is spin chromatic!

• Partial Snake concept works at magic energies: $1.76, \ 2.2, \ 2.64, \ ... \ \text{GeV}$. Saves the needed longitudinal field integral.

• At $7 \ \text{GeV}$ the HERA-like spin rotator approach looks most favorable. Spin transparent solution with all positive horizontal bends! But vertical bends contribute substantially to the vertical emittance growth.

• In both scenarios the average polarization of the circulated electron beam could reach $\zeta > 80\%$